Truthful Mechanisms for Scheduling Problems

Project B16: Mechanisms for Network Design Problems

Guido Schäfer

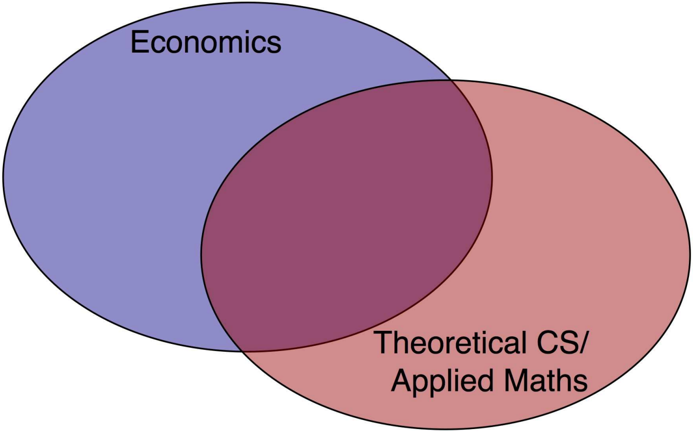
Technische Universität Berlin

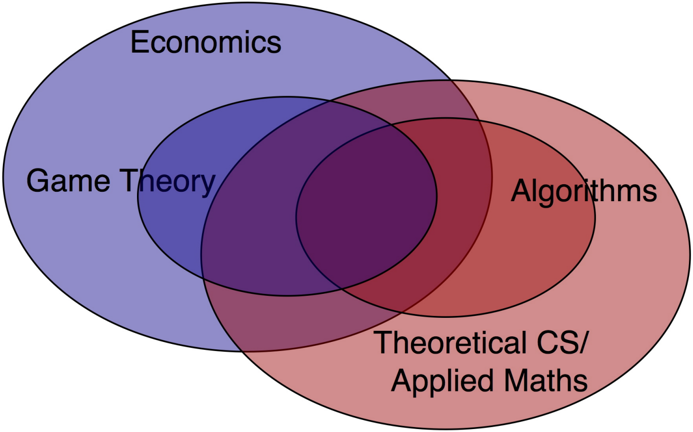
joint work with Janina Brenner

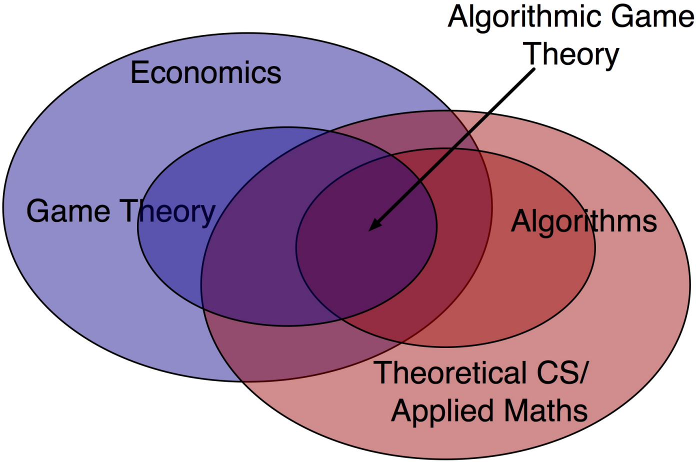
DFG Research Center MATHEON
Mathematics for key technologies

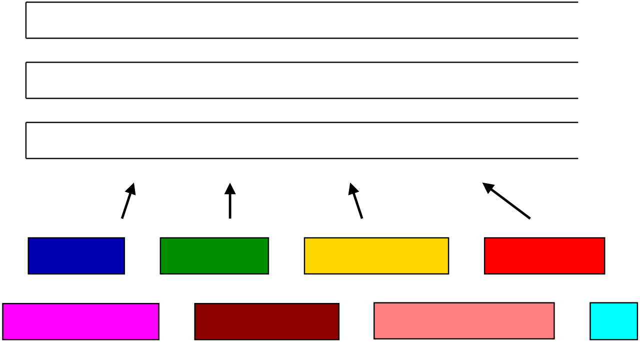
ICM/MATHEON Workshop, Warsaw, February 21–22, 2008

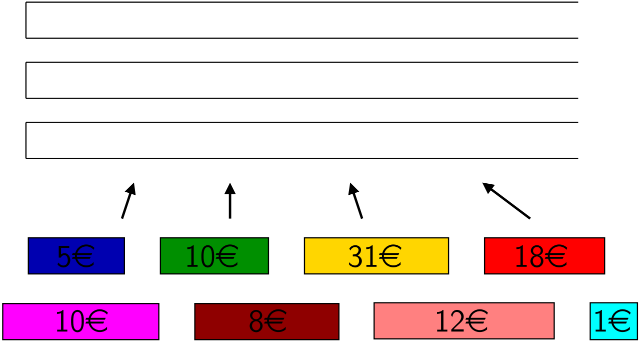
Algorithmic Game Theory

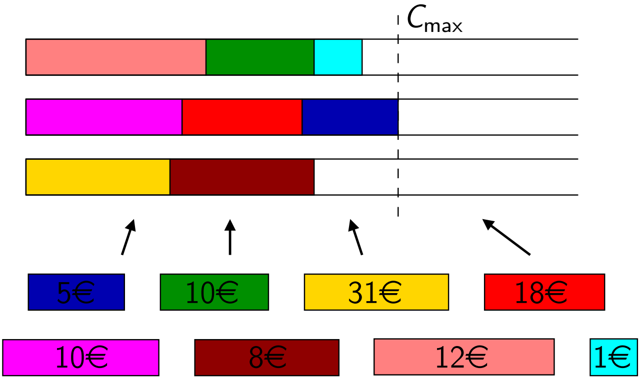


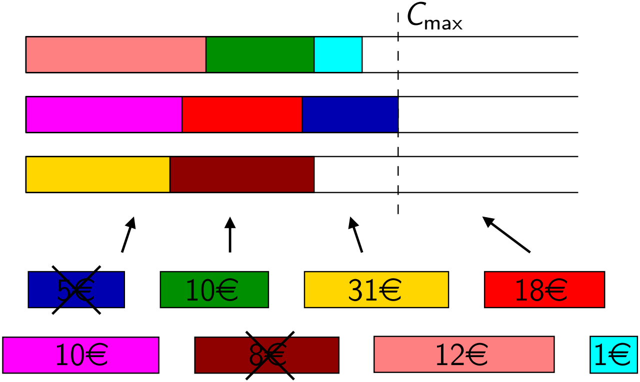

Economics
Game Theory
Algorithms
Theoretical CS/
Applied Maths

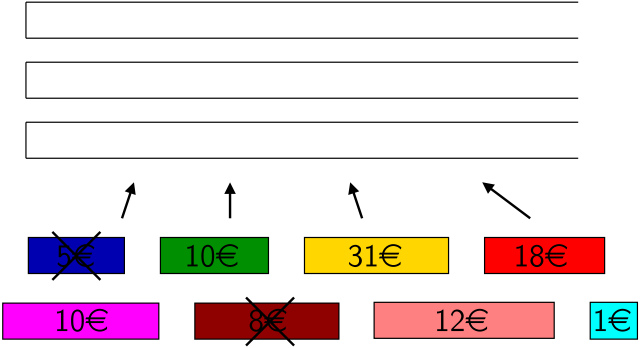

Algorithmic Game Theory
Economics
Game Theory
Algorithms
Theoretical CS/
Applied Maths

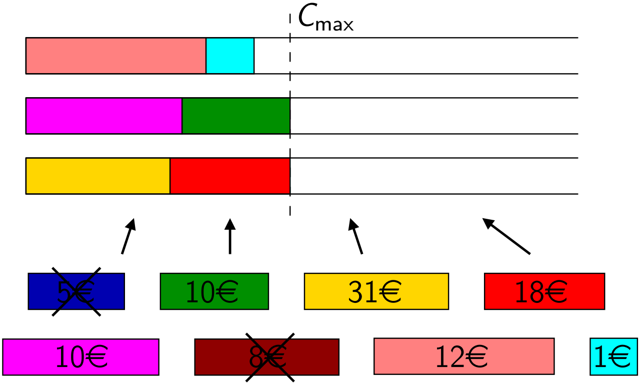



5€
10€
31€
18€
10€
8€
12€
1€


$C_{\max}$
5€
10€
31€
18€
10€
8€
12€
1€


$C_{\max}$
5€
10€
31€
18€
10€
8€
12€
1€


~~5€~~
10€
31€
18€
10€
~~8€~~
12€
1€


C_{max}
~~5€~~
10€
31€
18€
10€
~~8€~~
12€
1€

1. Cost sharing model, objectives, truthfulness
2. Moulin mechanisms
   - mechanism
   - state of affairs
   - limitations and new trade-offs
3. Singleton mechanisms
   - mechanism
   - framework: approximation algorithm $\longrightarrow$ singleton mechanism
   - three example applications
4. Concluding Remarks

Setting:

- service provider offers some service
- set U of potential users, interested in service
- cost function $C : 2^U \to \mathbb{R}^+$
  $C(S) =$ optimal cost to serve user-set $S \subseteq U$
- every user $i \in U$:
  - has a (private) valuation $v_i \geq 0$ for receiving the service
  - announces bid $b_i \geq 0$, the maximum amount he is willing to pay for the service

Cost sharing mechanism M:

- collects all bids $(b_i)_{i \in U}$ from users
- based on these bids:
  - decides a set $Q \subseteq U$ of users that receive service
  - computes (approximate) solution for Q of cost $\bar{C}(Q)$
  - determines a cost share $\xi_i(Q) \leq b_i$ for every user $i \in Q$

Cost sharing mechanism M:

- collects all bids $(b_i)_{i \in U}$ from users
- based on these bids:
  - decides a set $Q \subseteq U$ of users that receive service
  - computes (approximate) solution for Q of cost $\bar{C}(Q)$
  - determines a cost share $\xi_i(Q) \leq b_i$ for every user $i \in Q$

Strategic behaviour: every user $i \in U$ acts selfishly and attempts to maximize his utility:

- utility $u_i := v_i - \xi_i(Q)$ if served, $u_i := 0$ otherwise
- user manipulates mechanism if advantageous by misreporting his valuation, i.e., $b_i \neq v_i$

Strategyproofness: utility of every user $i \in U$ is maximized if he bids truthfully $b_i = v_i$, independently of other users

Strategyproofness: utility of every user $i \in U$ is maximized if he bids truthfully $b_i = v_i$, independently of other users

Group-strategyproofness: same holds true even if users form coalitions and coordinate their biddings

Illustration: Group-strategyproofness

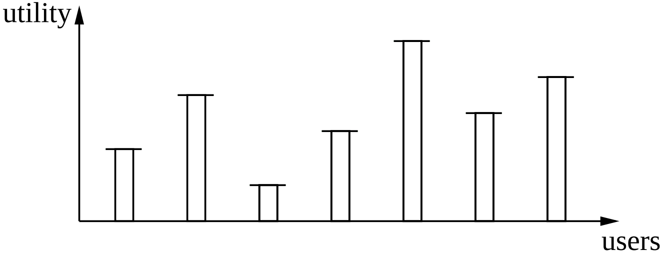


Illustration: Group-strategyproofness

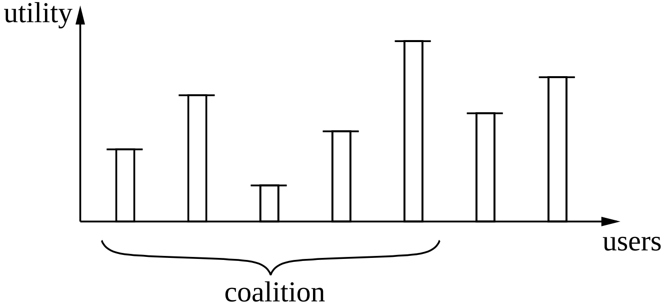


Illustration: Group-strategyproofness

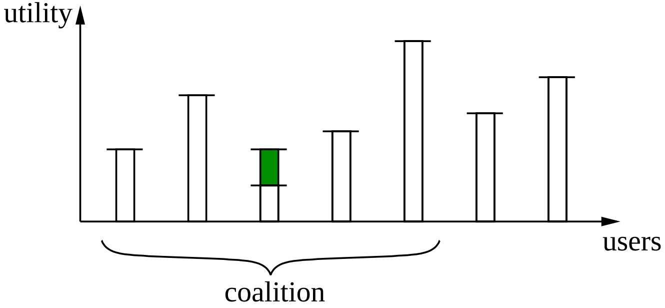
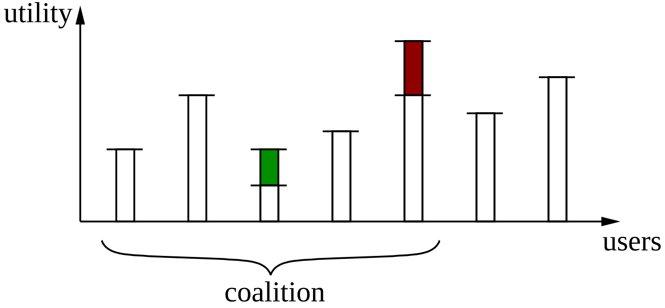


Illustration: Group-strategyproofness



β-budget balance: cost shares approximate servicing cost

$$\bar{C}(Q) \leq \sum_{i \in Q} \xi_i(Q) \leq \beta \cdot C(Q), \quad \beta \geq 1$$

Objectives

β-budget balance: cost shares approximate servicing cost

$$\bar{C}(Q) \leq \sum_{i \in Q} \xi_i(Q) \leq \beta \cdot C(Q), \quad \beta \geq 1$$

Social cost: define minimum social cost

$$\Pi^* := \min_{S \subseteq U} \Big\{ \sum_{i \notin S} v_i + C(S) \Big\}$$

β-budget balance: cost shares approximate servicing cost

$$\bar{C}(Q) \leq \sum_{i \in Q} \xi_i(Q) \leq \beta \cdot C(Q), \quad \beta \geq 1$$

Social cost: define minimum social cost

$$\Pi^* := \min_{S \subseteq U} \left\{ \sum_{i \notin S} v_i + C(S) \right\}$$

α-approximate: computed solution approximates social cost

$$\sum_{i \notin Q} v_i + \bar{C}(Q) \leq \alpha \cdot \Pi^*, \quad \alpha \geq 1$$

Moulin mechanism $M(\xi)$:

1: Initialize: $Q \leftarrow U$
2: If for each user $i \in Q$: $\xi_i(Q) \leq b_i$ then STOP
3: Otherwise, remove from Q all users with $\xi_i(Q) > b_i$ and repeat

Moulin mechanism $M(\xi)$:

1: Initialize: $Q \leftarrow U$
2: If for each user $i \in Q$: $\xi_i(Q) \leq b_i$ then STOP
3: Otherwise, remove from Q all users with $\xi_i(Q) > b_i$ and repeat

Thm: If ξ is cross-monotonic and β-budget balanced, then the Moulin mechanism $M(\xi)$ is group-strategyproof and β-budget balanced.

[Moulin, Shenker '01]
[Jain, Vazirani '01]

Upper bounds		β
[Moulin, Shenker '01]	submodular cost	1
[Jain, Vazirani '01]	minimum spanning tree	1
	Steiner tree and TSP	2
[Pal, Tardos '03]	facility location	3
	single-commodity rent-or-buy	15
[Leonardi, S. '03], [Gupta et al. '03]	single-commodity rent-or-buy	4
[Leonardi, S. '03]	connected facility location	30
[Könemann, Leonardi, S. '05]	Steiner forest	2
[Gupta et al. '07]	price-collecting Steiner forest	3
[Bleischwitz, Monien '07]	makespan scheduling	2

Lower bounds		β
[Immorlica et al. '05]	set cover, edge cover	$\Omega(n)$
	facility location	3
[Könemann et al. '05]	Steiner forest	2
[Bleischwitz, Monien '07]	makespan scheduling	2
[Brenner, S. '07]	completion time scheduling, etc.	$\Omega(n)$

Moulin Mechanisms: Known Results II

		β	α
[Roughgarden, Sundararajan '06]	submodular cost	1	$\Theta(\log n)$
	Steiner tree	2	$\Theta(\log^2 n)$
[Chawla, Roughgarden, Sundararajan '06]	Steiner forest	2	$\Theta(\log^2 n)$
[Roughgarden, Sundararajan '07]	facility location	3	$\Theta(\log n)$
	SRoB	4	$\Theta(\log^2 n)$
[Gupta et al. '07]	price-collecting SF	3	$\Theta(\log^2 n)$
[Brenner, S. '07]	makespan scheduling	2	$\Theta(\log n)$
	cost-stable problems		$\Omega(\log n)$

Limitations and New Trade-offs

Group-strategyproofness:

- very strong notion of truthfulness
- often bottleneck in achieving good performance guarantees
- strong lower bounds exist, even if we allow exponential time

Group-strategyproofness:

- very strong notion of truthfulness
- often bottleneck in achieving good performance guarantees
- strong lower bounds exist, even if we allow exponential time

Idea: use (slightly) weaker notion of group-strategyproofness:
weak group-strategyproofness [Mehta et al. '07]

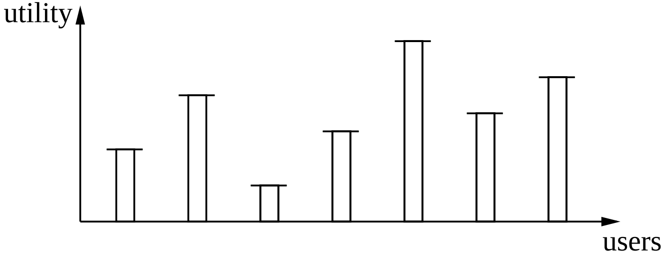
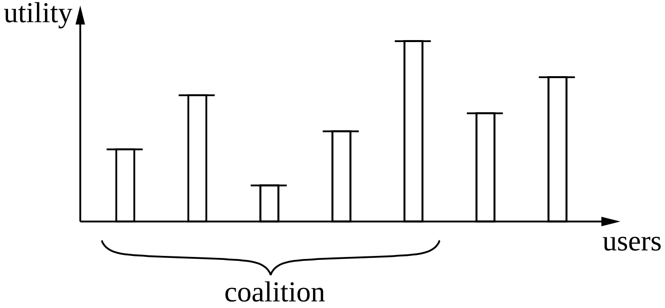
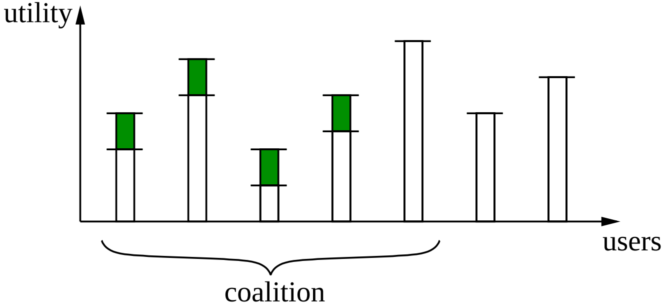

utility
users

Illustration: Weak Group-strategyproofness




utility
users
coalition

Offer function: Let $\tau : U \times 2^U \rightarrow \mathbb{R}^+$ be an offer function
$\tau(i, S) =$ offer time of user i with respect to $S \subseteq U$

Offer function: Let $\tau : U \times 2^U \rightarrow \mathbb{R}^+$ be an offer function
$\tau(i, S) =$ offer time of user i with respect to $S \subseteq U$

Singleton offer function: for every subset $S \subseteq U$ and for every two users $i, j \in S$: $\tau(i, S) \neq \tau(j, S)$

Singleton Mechanisms

Offer function: Let $\tau : U \times 2^U \to \mathbb{R}^+$ be an offer function

$\tau(i, S) =$ offer time of user i with respect to $S \subseteq U$

Singleton offer function: for every subset $S \subseteq U$ and for every two users $i, j \in S$: $\tau(i, S) \neq \tau(j, S)$

Singleton mechanism $M(\xi, \tau)$:

1: Initialize: $Q \leftarrow U$

2: If for each user $i \in Q$: $\xi_i(Q) \leq b_i$ then STOP

3: Otherwise: Among all users in S with $\xi_i(S) > b_i$, let i^* be the one with minimum offer time $\tau(i, S)$. Remove i^* from Q and repeat.

Thm: Let ALG be a ρ-approximation algorithm that satisfies certain conditions. Then ALG can be turned into a singleton mechanism that is weakly group-strategyproof and ρ-budget balanced.

[Brenner, S. '08]

Consistent singleton offer function: for every $T \subseteq S$:

S	1	2	3	4	5	6	7	8	9	($\tau(\cdot, S)$ order)
T	1	2	3		5	6		8	9	($\tau(\cdot, S)$ order)

Consistency & τ-increasing

Consistent singleton offer function: for every $T \subseteq S$:

S	1	2	3	4	5	6	7	8	9	($\tau(\cdot, S)$ order)
T	1	2	3		5	6		8	9	($\tau(\cdot, S)$ order)

Consistency & τ-increasing

Consistent singleton offer function: for every $T \subseteq S$:

S	1	2	3	4	5	6	7	8	9	($\tau(\cdot, S)$ order)
T	1	2	3		5	6		8	9	($\tau(\cdot, S)$ order)
T	1	2	3	9	8	5	6			($\tau(\cdot, T)$ order)

Consistency & τ-increasing

Consistent singleton offer function: for every $T \subseteq S$:

S	$\boxed{1\ 2\ 3}$ 4 5 6 7 8 9	($\tau(\cdot, S)$ order)
T	1 2 3 ● 5 6 8 9	($\tau(\cdot, S)$ order)
T	$\boxed{1\ 2\ 3}$ 9 8 5 6	($\tau(\cdot, T)$ order)

τ-increasing: ALG is τ-increasing if for every $S \subseteq U$ and every $i \in S$:

$$\xi_i(S) := \bar{C}(S_i) - \bar{C}(S_{i-1}) \geq 0,$$

where S_i is the set of the first i elements of S (ordered according to $\tau(\cdot, S)$).

Example I

Problem: parallel machines, no preemption, minimize makespan

Offer function: order jobs by non-increasing processing times (Graham's rule)

Thm: There is a singleton mechanism that is weakly group-strategyproof and 4/3-budget balanced.

Contrast: lower bound for Moulin mechanisms: 2 (budget balance)

Problem: parallel machines, no preemption, minimize sum of weighted completion times

Offer function: order jobs by non-increasing weight per processing time (Smith's rule)

Thm: There is a singleton mechanism that is weakly group-strategyproof, 1.21-budget balanced, and 2.42-approximate.

Contrast: lower bound for Moulin mechanisms: $\Omega(n)$ (budget balance)

Problem: single machine, release dates, preemption, minimize sum of completion times

Offer function: order jobs by increasing completion times in the shortest remaining processing time schedule

Thm: There is a singleton mechanism that is weakly group-strategyproof, 1-budget balanced, and 4-approximate.

Contrast: lower bound for Moulin mechanisms: $\Omega(n)$ (budget balance)

Concluding Remarks

- developed framework to convert approximation algorithms into weakly group-strategyproof mechanisms (if only some mild conditions are satisfied)

Concluding Remarks

- developed framework to convert approximation algorithms into weakly group-strategyproof mechanisms (if only some mild conditions are satisfied)
- slightly relaxed notion of truthfulness leads to significant improvement in performance guarantees; still allowing coalitions

Concluding Remarks

- developed framework to convert approximation algorithms into weakly group-strategyproof mechanisms (if only some mild conditions are satisfied)
- slightly relaxed notion of truthfulness leads to significant improvement in performance guarantees; still allowing coalitions
- framework yields good approximation ratios in scheduling context

Concluding Remarks

- developed framework to convert approximation algorithms into weakly group-strategyproof mechanisms (if only some mild conditions are satisfied)
- slightly relaxed notion of truthfulness leads to significant improvement in performance guarantees; still allowing coalitions
- framework yields good approximation ratios in scheduling context
- by-product: obtain approximation algorithms for respective scheduling problems with rejection

Concluding Remarks

- developed framework to convert approximation algorithms into weakly group-strategyproof mechanisms (if only some mild conditions are satisfied)
- slightly relaxed notion of truthfulness leads to significant improvement in performance guarantees; still allowing coalitions
- framework yields good approximation ratios in scheduling context
- by-product: obtain approximation algorithms for respective scheduling problems with rejection

Thank you!